ESTIMATING RATES OF VALUE CHANGE FROM DATA VALUES

Estimating rate of change from a series of asset values

A series of year-by-year value data may be available to estimate the rate of change in value of a type of asset. The series could be, say, from market values obtained in deep and well-established secondary markets or from a taxpayer's history of sale values over different periods of use under conditions peculiar to the taxpayer.

Regression analysis packages or functions – such as those available in Microsoft Excel – may be used to determine the exponential curve that best 'fits' the series of year-by-year values over the period of analysis. That exponential 'line of best fit' is represented by:

$$y = b.m^{t}$$

where
- y = 'fitted' value of asset over time
- b = 'fitted' constant which equals the 'fitted' value of the asset at the start of the first year (t=0)
- m = 'fitted' exponential coefficient
- t = the series of years with corresponding actual asset values (for example, 0,1,2….7,8).

This exponential function simply says that, commencing with the initial 'fitted' value of the asset, b, at the start of the Year 1: the 'fitted' value of the asset at the end of Year 1 is b.m; the 'fitted' value at the end of Year 2 is $b.m^2$: the 'fitted' value at the end of Year 3 is $b.m^3$; and so on.

When asset value is declining year by year, m = 1 – rate of decline.

Thus, the constant rate of decline, r, estimated by this 'line of best fit' is determined from the identity:

$$r = m – 1$$

Thus, if the exponential coefficient, m, from the 'line of best fit' is 0.750 the estimated constant rate of decline in value is 0.25. With depreciable assets, the 0.25 rate of decline is mirrored in a corresponding declining balance rate.

Adjusting for one-off effects

Account may need to be taken of one-off changes in say technological advances or taste changes occurring in particular years so these one-off events do not bias the estimate of the future rate of value decline. 'Dummy' variables set in recognition of one-off effects in particular years – and possibly in recognition of lasting impact of such effects – could be used in a multivariate regression analysis to handle such circumstances.

Estimating rate of change from two data values

Sometimes there may be just two data points from which to estimate rate of change in value of a particular asset: initial purchase price and ultimate sale price in a subsequent year. Following is a method of calculating the rate of change in these circumstances.

The relationship between initial value, A, and subsequent value, V, is given by the standard compounding formula:

$$V = A \,.\, (1 + r)^{T}$$

where r = annual rate of change in value (negative for declining values)
T = number of years between initial and final value.

Taking logs (to the base e) of both sides,

$$\ln V = \ln A + T \,.\, \ln (1 + r)$$

So,

$$\ln (1+r) = (\ln V - \ln A)/T$$

Translating into exponential notation and rearranging,

$$r = e^{(\ln V - \ln A)/T} - 1$$

To illustrate the use of this expression for the annual rate of change in value, take an asset with an initial value (A) of $29,610.0, which declines over 4 years to $14,532.5 (V). From these data, mathematical/financial calculators can be used to compute the following:

$$\ln V = 9.58$$

$$\ln A = 10.30$$

$$(\ln V - \ln A)/T = (9.58 - 10.30)/4 = -0.1779$$

$$e^{(\ln V - \ln A)/T} = 0.8370$$

$$r = -0.1630$$

Thus, the estimated annual constant rate of decline in value is 16.3 per cent.

Adjusting estimated rates of decline for general inflation

The underlying rate of non-inflationary decline in value of a depreciating tangible asset is affected by inflation. With non-inflationary decline in value at the rate s and zero inflation, a depreciable asset's value, say V, in one year declines to (1-s)V the next. This means that the asset's value declines each year by the proportion (1-s) – 1 or just –s.

With inflation at the rate i rather than zero, the asset's value V in one year would change to (1-s)(1+i)V the next. Its value would therefore be changing each year by the proportion (1-s)(1+i) – 1. Thus, for an asset whose value declines at 25 per cent with no inflation, with inflation at 5 per cent, value should decline by the proportion (.75)(1.05) – 1 or 0.2125. That is, the asset's value declines by 21.25 per cent per year – rather than the 25 per cent decline with no inflation. Applying the estimated 21.25 per cent declining balance write-off would seee depreciation allowances matching the after-inflation decline in asset value. Were declining balance write-off applied at 25 per cent during years when 5 per cent inflation occurred, the depreciation allowances would be too generous in present value terms.

Asset data values over time from which a constant rate of decline is estimated for future income tax depreciation purposes should include a level of inflation that is in line with expected future inflation. Adjustments for inflation are required when asset value data used is collected when inflation is at a different level to that expected in future or includes variable levels of inflation caused say by one-off events.

Deflate data and adjust for expected inflation or use 'dummy' variables

A way of removing the effects of past variable inflation, which could be applied in most circumstances, would be to deflate the original asset value data using a suitable price index series. The rate of decline in value estimated from the deflated data would correspond with the non-inflationary rate of decline (25 per cent rate in the above example). That non-inflationary rate of decline would then need to be adjusted to incorporate the level of inflation expected in future when the declining balance depreciation rates are to be applied. With 2 ½ per cent inflation assumed (consistent with the Reserve Bank of Australia's objectives), a 25 per cent non-inflationary rate of decline becomes:

$$\text{Adjusted nominal rate of decline} = (1 - 0.25)(1 + 0.025) - 1$$

$$= 0.23$$

With 2 ½ per cent inflation expected in future, the 25 per cent non-inflationary decline in value is reduced to 23 per cent.

Alternatively, 'dummy' variables for selected years could be used in a multivariate regression analysis to estimate rate of decline that recognises one-off effects (say because of changed indirect tax arrangements) or a number of years with inflation at a broadly constant level but differing from future expectations of inflation.

Adjustment when inflation is different, but constant, in estimation period

Where inflation is say higher (and broadly constant) over the whole estimation period and lower (and again broadly constant) in the years when depreciation is to be applied, an adjustment can be made directly from the rates of inflation applying in the two periods.

To illustrate, take the previous example of an asset with a 16.3 per cent rate of decline estimated from two data points: initial purchase price and subsequent sale value after 4 years. Assume that the level of inflation occurring over the 4 years was 5 per cent per annum but that inflation is expected to be 2 ½ per cent in the future when the results of the estimation are to determine declining balance depreciation rates.

Substituting the estimate of the rate of value decline and the inflation rate applying over the estimation period into the formula for the rate of decline under inflation,

$$(1-s)(1 + 0.05) - 1 = -0.1630$$

Therefore,

$$s = 0.2029$$

Thus, the non-inflationary rate of deterioration implied from the estimated rate of decline in value is 20.3 per cent per year.

Applying this value of s along with the 2 ½ per cent inflation level expected in the future provides the inflation-adjusted rate of decline:

$$(1-0.203)(1 + .025) - 1 = -0.183$$

The rate of decline to be used for declining balance write-off is 18.3 per cent. The assumed reduction in inflation from 5 to 2 ½ per cent has increased the rate of declining balance write-off by about 2 percentage points.

The higher rate of decline in asset value under lower inflation would result in a $13,200 value for the asset after 4 years (rather than the $14,532.5 under 5 per cent inflation). Using the lower residual value of the asset to further illustrate the use of the methodology for determining annual rate of decline in value from just two data points:

$$\ln V = 9.49$$

$$\ln A = 10.30$$

$$(\ln V - \ln A)/t = (9.49 - 10.30)/4 = -0.2020$$

$$e^{(\ln V - \ln A)/t} = 0.8171$$

$$r = -0.1829$$

As expected, the estimated annual constant rate of decline in value is 18.3 per cent.